THE FINAL PROOF SUPPORTING THE TURNOVER FORMULA.

I would like to thank Aris for his mathematical contributions and his "sweat" which has enabled a deeper understanding of the turnover formula to emerge. His contribution needs to be recognized.

In his first contribution he modelled the formula-in-general where all sales tended towards a common average, proving its accuracy. By beginning with the general, we were following Marx's method of investigation which begins with the abstract (the pure forms) before proceeding to the concrete. Only through equal exchange, a necessary abstraction, was Marx able to prove the connection between value and price. Similarly, had Aris's modelling proved the formula to be inaccurate in its uncomplicated form, then clearly it would have had to be discarded.

The real question is how we move from the abstract to the concrete thereby proving the formula in real life. Why it is the case that no matter what independent proof has been provided, it always confirms the accuracy of the formula, provided of course the national data on which it is based is robust. To move to the concrete, I asked Aris to model the turnover formula using <u>unaverage</u> sales. The results appear in the paper he presented below.

The importance of this paper lies in part 1 rather than part 2 (in Table 1 rather than Table 2). The reasons will become clear soon. Part 1 shows the effect on turnover of changes to the weighting of the final sale (sale 10). It shows that the higher the weight, the slower the turnover. The number of turnovers referred to here are highlighted in yellow in Table 1. At first glance this seems to contradict the formula. How can ten turnovers suddenly become 9 when the weight of the final sale is 20% rather than 10%? The answer is because concretely the turnover was always 9. If the weight of intermediate sales is 80% and that of final sales is 20%, then turnover from the outset would have been 9 instead of a hypothetical 10.

The confirmation is provided by the manner in which the statistical bureaus construct their system of National Accounts (SNA). The SNA is organized by industry. What this means is that the sum of final sales is in fact the final sales for a particular industry. Take the "Motor vehicles, bodies and trailers, and parts" industry listed under durable manufacturing. The final sales here add up to the annual sales (less inventory adjustments) of motor vehicles and parts to final consumers (drivers), dealers, leasing companies, repair shops, and so on.

What is important is the weight of these sales relative to the gross output for this industry. The smaller the weight, the greater must be the weight of intermediate inputs. Over the last 30 years there has been a major restructuring of this industry. The actual car companies have divested much of their manufacturing base and reduced themselves more or less to assemblers of cars. Much of what they produced in-house they now purchase from the international automotive chains. The result is that the weight of their final sales as a share of the gross output of the car industry has shrunk. On the other hand, the weight of intermediate sales has grown.

The result is that turnover has accelerated from an annual rate of 6.7 in 1997 to 8.3 in 2017, or conversely, the circulating period has reduced from 54.5 days to only 44 days. These results are confirmed by actual and reported changes in the industry itself. In other words, the turnover formula is capturing the dynamic of this industry and the changing ratios between intermediate sales and final sales.

If we examine GM's balance sheet for the 9 months ended 30th September 2017, and, use its revenue of $98,983 million as gross output and the sum of depreciation, sga costs (for wages) plus operating income to arrive at GVA we end up with a turnover of 8.1. ((https://investor.gm.com/static-files/8d51e91f-9008-4208-a61e-aed1ef925fef) The closeness in turnover occurs despite the fact that GM does not release actual compensation figures forcing the use of general, administrative and selling expenses as a substitute or proxy. Furthermore, this acceleration in turnover is always associated with a reduction in working capital. A glance at GM's balance sheet shows that it not only has reduced its working capital but offloaded part of it onto suppliers, as credit received exceeds credit given by 300%.

Which brings me to the second part of Aris's modelling. Here he examines variations in sales number 1 – 9 (in Table 2). Now it is important to note that sales 1 – 9 add up to the sum of intermediate sales. These sales generally belong to the output of other industries. Some of them will constitute final sales in those industries. For example, the power used in the "Manufacturing" industry would constitute a final sale taken in the "Utilities" industry. It is for this reason that the variation found in inputs is unimportant. What is important is the aggregation of these inputs to determine the ratio of these inputs relative to the final sales of the industry under investigation. We remain unconcerned with the individual variations within this aggregate total.

To repeat, as far as the formula is concerned, what is important is the share of intermediate sales and the share of final sales in Gross Output. The greater the share of intermediate sales the greater will be the distance between gross output and gross value added, the faster turnover and vice versa. This is the general law. It is what the turnover formula captures. And that is why it is accurate. That is why it can be verified by independent proofs because they are measuring the same thing - the turnover that is characteristic of that industry.

Had it not been for Aris's hard work and the communications between us I may have missed the essential point that final sales were industry sales to other industries or end users. So the formula is measuring sales coming into the industry versus sales leaving the industry, together with the difference in the value of these sales because of the value added by the industry itself. This is reflected in the two-part nature of the formula.

Brian Green.

The Empirical Turnover Equation's variance in one term

Da Cunha Da Costa Dias Aris

In this article we will examine Green's empirical turnover equation's[i] variance in one term.
We have:

$$I \; ; \; \frac{GO}{GVA} + \frac{GO - GVA}{GVA} = \frac{2 \cdot GO - GVA}{GVA} \quad (1),$$

where I is the approximate number of turnovers per time unit, GO is gross output and GVA is gross value added.

An Example

Following Green ([ii]), we consider 10 industries, each one's sales used as input of the next one. For industries #1 to #9, value added is α and for industry #10 value added is b. Input equals $(i-1)\alpha$ for industry #i, while the value of sales is $i \cdot \alpha$ for industries #1 to #9 and $9\alpha + b$ for industry #10.
Input plus value added equals value of sales. The total sum of value added is GVA and total sales are GO.

	Input	+Value added	=Value of Sales
1st industry	0	α	a
2nd industry	a	α	$2a$
3rd industry	$2a$	α	$3a$
4th industry	$3a$	α	$4a$
5th industry	$4a$	α	$5a$
6th industry	$5a$	α	$6a$
7th industry	$6a$	α	$7a$
8th industry	$7a$	α	$8a$
9th industry	$8a$	α	$9a$
10th industry	$9a$	b	$9a+b$
Total (Sums)	$45a$	$9a+b$	$54a+b$

From equation (1) we have:

$$I = \frac{2 \cdot GO - GVA}{GVA} \Rightarrow I = \frac{2 \cdot 54a + 2b - 9a - b}{9a + b} \Rightarrow I = \frac{99a + b}{9a + b}$$

Let us vary b, attributing to it k% of the total value added.

$$b = k \cdot \mathrm{GVA} \Leftrightarrow \mathrm{GVA} = \frac{b}{k} = 9a + b \Leftrightarrow 9a = b\left(\frac{1}{k} - 1\right) \Leftrightarrow b = \frac{9ka}{1-k}$$

So we have $I = \dfrac{99a + \dfrac{9ka}{1-k}}{9a + \dfrac{9ka}{1-k}} = \dfrac{99(1-k)+9k}{9(1-k)+9k} \Rightarrow I = \dfrac{99-90k}{9}$

We vary b, and calculate the number of turnovers I:

Table 1.

b (%GVA)	**b** (in units of *a*)	**I**
5%	0,47	10,5
10%	1	10
15%	1,59	9,5
20%	2,25	9
25%	3	8,5
30%	3,86	8
35%	4,85	7,5
40%	6	7
45%	7,36	6,5
50%	9	6
55%	11	5,5
60%	13,5	5
65%	16,71	4,5
70%	21	4
75%	27	3,5
80%	36	3
85%	51	2,5
90%	81	2
95%	171	1,5

We also have for $b \to 100\%\text{GVA}$ that $I \to 1$ and for $b \to 0\%\text{GVA}$ that $I \to 11$.

Variance in one term

Let's consider the general case where only one term varies. So, let every industry's value added be α, except the *i*th for which value added is *b*. Let the number of industries preceding it be *n* and following it *m*.

	Input	+Value added	=Value of Sales
1st industry	0	α	a
2nd industry	a	α	$2a$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
nth industry	$(n-1)a$	α	na
(n+1)th industry	na	b	$na+b$
(n+2)th industry	$na+b$	α	$na+b+a$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
(n+m+1)th industry		α	$na+b+ma$
Total (Sums)		$na+ma+b$	

Total value of sales (equaling GO) is $na + mna + (m+1)b + a\left(\sum_{i=1}^{n} i + \sum_{j=1}^{m} j\right)$

So we get

$$I = \frac{2 \cdot GO - GVA}{GVA} \Rightarrow I = \frac{2 \cdot \left(na + mna + (m+1)b + a\left(\sum_{i=1}^{n} i + \sum_{j=1}^{m} j\right)\right) - (na + ma + b)}{na + ma + b}$$

$$\Rightarrow I = \frac{na + m(2n-1)a + (2m+1)b + 2a\left(\sum_{i=1}^{n} i + \sum_{j=1}^{m} j\right)}{a(n+m) + b}$$

Thus for b ? a we can see that:

- As $m \to 0$ (and $\frac{a}{b} \to 0$) $\lim_{\substack{m \to 0 \\ a/b \to 0}} I = 1$
- As $n \to 0$ (and $\frac{a}{b} \to 0$) $\lim_{\substack{n \to 0 \\ a/b \to 0}} I = 2m+1$
- As $m \approx n$ (and $\frac{a}{b} \to 0$) $\lim_{\substack{m \to n \\ a/b \to 0}} I = 2n+1 = 2m+1 = m+n+1$

From the above we can discern the turnover equation's dependence on the position of *b*. If the dominant factor of value production is the last industry, the turnovers tend to 1. If it is the first industry then the turnovers are nearly double that of a totally homogenous distribution of value added ($b = a$). Finally, if the predominant industry is in the middle of the distribution then the turnover changes little from the totally homogenous case.

Variance in one term for ten industries.

So we repeat the previous calculations, varying *b* by 5% of GVA intervals, and changing its industry's position in the set.

Table 2.

	b (%GVA)	5%	10%	15%	20%	25%	30%	35%	40%	45%	50%
	b (units of a)	0,47	1	1,59	2,25	3	3,86	4,85	6	7,36	9
b's position											
1	**I**	**9,50**	**10,00**	**10,50**	**11,00**	**11,50**	**12,00**	**12,50**	**13,00**	**13,50**	**14,00**
2		**9,61**	**10,00**	**10,39**	**10,78**	**11,17**	**11,56**	**11,94**	**12,33**	**12,72**	**13,11**
3		**9,72**	**10,00**	**10,28**	**10,56**	**10,83**	**11,11**	**11,39**	**11,67**	**11,94**	**12,22**
4		**9,83**	**10,00**	**10,17**	**10,33**	**10,50**	**10,67**	**10,83**	**11,00**	**11,17**	**11,33**
5		**9,94**	**10,00**	**10,06**	**10,11**	**10,17**	**10,22**	**10,28**	**10,33**	**10,39**	**10,44**
6		**10,06**	**10,00**	**9,94**	**9,89**	**9,83**	**9,78**	**9,72**	**9,67**	**9,61**	**9,56**
7		**10,17**	**10,00**	**9,83**	**9,67**	**9,50**	**9,33**	**9,17**	**9,00**	**8,83**	**8,67**
8		**10,28**	**10,00**	**9,72**	**9,44**	**9,17**	**8,89**	**8,61**	**8,33**	**8,06**	**7,78**
9		**10,39**	**10,00**	**9,61**	**9,22**	**8,83**	**8,44**	**8,06**	**7,67**	**7,28**	**6,89**
10		**10,50**	**10,00**	**9,50**	**9,00**	**8,50**	**8,00**	**7,50**	**7,00**	**6,50**	**6,00**

	b (%GVA)	55%	60%	65%	70%	75%	80%	85%	90%	95%
	b (units of a)	11	13,5	16,71	21	27	36	51	81	171
b's position										
1	**I**	**14,50**	**15,00**	**15,50**	**16,00**	**16,50**	**17,00**	**17,50**	**18,00**	**18,50**
2		**13,50**	**13,89**	**14,28**	**14,67**	**15,06**	**15,44**	**15,83**	**16,22**	**16,61**
3		**12,50**	**12,78**	**13,06**	**13,33**	**13,61**	**13,89**	**14,17**	**14,44**	**14,72**
4		**11,50**	**11,67**	**11,83**	**12,00**	**12,17**	**12,33**	**12,50**	**12,67**	**12,83**
5		**10,50**	**10,56**	**10,61**	**10,67**	**10,72**	**10,78**	**10,83**	**10,89**	**10,94**
6		**9,50**	**9,44**	**9,39**	**9,33**	**9,28**	**9,22**	**9,17**	**9,11**	**9,06**
7		**8,50**	**8,33**	**8,17**	**8,00**	**7,83**	**7,67**	**7,50**	**7,33**	**7,17**
8		**7,50**	**7,22**	**6,94**	**6,67**	**6,39**	**6,11**	**5,83**	**5,56**	**5,28**
9		**6,50**	**6,11**	**5,72**	**5,33**	**4,94**	**4,56**	**4,17**	**3,78**	**3,39**
10		**5,50**	**5,00**	**4,50**	**4,00**	**3,50**	**3,00**	**2,50**	**2,00**	**1,50**

We can observe the previously mentioned general characteristics in this example of ten industries. When $b = a$, the number of turnovers is 10 (the homogenous case). If the predominant industry in terms of value added is at the beginning of the production chain, then the number of turnovers tends to the double of the homogenous case. If the predominant industry is at the middle of the production chain then the number of turnovers changes only a little. And, finally, if it is at the end of the production chain, the turnover become less than 10.

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[i] APPLYING THE TURNOVER FORMULA TO THE SYSTEM OF NATIONAL ACCOUNTS TO DETERMINE BOTH THE AMOUNT OF WORKING CAPITAL AND ITS ANNUAL RATE OF TURNOVER, Brian Green, 2017.

[ii] APPLYING THE TURNOVER FORMULA TO THE SYSTEM OF NATIONAL ACCOUNTS TO DETERMINE BOTH THE AMOUNT OF WORKING CAPITAL AND ITS ANNUAL RATE OF TURNOVER, Brian Green, 2017.