

AN OUTSTANDING EXPOSITION OF MARX'S TRANSFORMATION PROBLEM.

[IN DÉFENSE OF MARX](#) ON THE "TRANSFORMATION PROBLEM" A Response to Paul Sweezy and Anwar Shaikh, Part 1 by James Miller is a rare and very welcome post on his Substack site. When it comes to defending Chapter 9 in Volume 3, many Marxist scholars run for cover. Often times they overturn the method embodied in Chapter 9 which begins with value and moves on to price by instead beginning with price to remove any incongruities between inputs and outputs. But that defeats the whole purpose of the Chapter which is designed to reveal the elastic link between value and price.

Mr Miller outlines this particularly well: *"But the table does show the transformation of values into prices of production in a sufficiently comprehensible way. What would be the alternative? If you were to say that the constant and variable capitals making up the advanced capital should be shown as "already transformed" into prices of production, then there is nothing in the table but prices of production. Values are not represented at all. With only prices of production entered into the table, and no values, then the table does not demonstrate the transformation of the one to the other."* And again: *"How then, can it be said that Marx erred here? Bortkiewicz said that the error consists in listing the constant and variable capitals as values, when they should be listed as prices of production. But the purpose of the table is to show the difference between values and prices of production. How can Marx show this difference if he lists the constant and variable capitals as already transformed into prices of production? If this were to be done, then values would not appear at all in the table, and the difference between values and prices of production could not be demonstrated."* Bravo!

This failure is not limited to Von Bortkiewicz but is found in Marxist circles as well including those who push the TSSI (Temporal Single-System Interpretation) as well as those who take the aggregate value which obviously equals the aggregate of prices as their starting point, obscuring the fact that capitalism exists as many capitals which differ in their composition, and it is this which must serve as our starting point.

As Mr Miller says Marx did not invent this problem of price formation. Ricardo observed the paradox between value and price but was unable to solve it. For anyone interested in a brief background discussion around the problem of price, this article will suffice. It sets out the problem, the historical background and the solution arrived at by Marx.

However, the author like many others does not appreciate that Chapter 9 and 10 together deal with two, not one, transformations. Chapter 10 which should have been placed before Chapter 9 deals with the transformation of individual values into market values which formed the original market prices before the industrial revolution, in what was the period of manufacturers. Market value is the weighted average labour time needed to produce a commodity. Only this weighted average when multiplied by the quantity produced will equal the total labour expended on its production. Or as Marx puts it: *"[This mode of determining](#) market-values, which we have here outlined abstractly, is promoted in the real market by competition among the buyers, provided the demand is large enough to absorb the mass of commodities at values so fixed."* (Readers who visit my site will see this explained repeatedly due to its importance.)

It was the industrial revolution which in transforming the technical compositions of capital magnified the differences between industries, calling forth the necessity of prices of production to ensure market prices

lubricated the now more complex set of exchanges. The fact is that only market values can be transformed into prices of production, and had Engels structured Volume 3 on the basis of the historico-logic method, it would have been made clear that the tables found in Chapter 9 begin with capital at market values because Chapter 10 would have preceded it. "[*The exchange of commodities at their values, or approximately at their values, thus requires a much lower stage than their exchange at their prices of production, which requires a definite level of capitalist development.*](#)"

LADISLAUS VON BORTKIEWICZ

The author is at his best explaining the method of Von Bortkiewicz and its shortcomings. Grasping Von Bortkiewicz's method is not easy, but the author makes it easier by patiently explaining what this mathematician set out to do. Essentially, Von Bortkiewicz benevolently picked Marx up off the pavement only to throw him under a bus. He did not correct Marx but obscured what Marx had set out to do. Despite this, many 'disciples' of Marx have adopted elements of Von Bortkiewicz's procedures.

The author does an excellent job discussing how Chapter 9 condenses a historical process which took many decades before market prices of production displaced market prices based on market values. He criticises Von Bortkiewicz for failing to incorporate this reality by seeking to replace a historical process by a set of mathematical formulae based on incomplete assumptions.

Where I disagree with the author is that it is impossible to transform market values into market prices of production once the composition of capital is detailed, the rate of exploitation is provided as is the mass of value produced. The reason for this possibility is found in Chapter 9 itself. There Marx describes the direction in which surplus value must flow and the extent of this flow. Direction and quantity make up a vector." [*Vector, in physics, a quantity that has both magnitude and direction. It is typically represented by an arrow whose direction is the same as that of the quantity and whose length is proportional to the quantity's magnitude.*](#)" All competition does is execute this inner law when it harmonises profitability through the movement of capital. Competition may appear to be the product of invisible hands but in the end it is constrained by the inner laws which regulate it as identified by Marx in Chapter 9.

The author criticises Von Bortkiewicz for misusing the Tables found in Volume 2 of capital as the basis of his analysis. Here I differ from the author. It is possible in all circumstances to convert values into prices and vice versa if there is sufficient information to set out the vectors. This is what I have done below based on these same tables. But before preceding I would like to add in an assumption that has lain hidden ever since *Das Kapital* was published, and it is this. In Chapter 9 Marx uses an undivided stream of surplus value to harmonise profit. This is fine for Marx's partial elaboration of the transformation problem, really no more than an arithmetical example of how much and in which direction, surplus value must be redistributed to achieve the profit equalisation for 5 known but differing capitals.

The fact that Marx's tables are merely arithmetical examples demonstrating the vectors (in modern parlance) can be found in a number of ways. The author quotes Marx's own cautionary note. "*We had originally assumed that the cost price of a commodity equaled the value of the commodities consumed in its production. But for the buyer, the price of production of a specific commodity is its cost price, and may thus pass as cost price into the prices of other commodities. Since the price of production may differ from the value of a commodity, it follows that the cost price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it. It is necessary to remember this modified significance of the*

cost price, and to bear in mind that there is always the possibility of an error if the cost price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point.” (Marx, 1962, p. 162) A second example is the terminology used by Marx in Chapter 9 where he refers to commodity-price not price of production, a term which signifies that he is dealing with a hybrid – neither pure value nor a finalised market price.

But the real proof that Marx is merely giving an example lies in the fact that when the capitalists withdraw their share of the pool of surplus value amounting to 110 in total, then the amount of capital reverts back to their original values of $100 \times 5 = 500$. This can be seen in the Table below taken from [my original article on the transformation problem](#). There we see when each capitalist withdraws their 22 in profit to be consumed unproductively it reinstates the original value of each capital at 100. We are therefore back where we started with the example, and it can be repeated ad nauseum and it wont alter the end result.

Table 7.

Capital	Price of Commodity	less profit	equals c+v	plus unconsumed capital	Equals Total Capital.
1	92	22	70	30	=100
2	103	22	81	19	=100
3	113	22	91	9	=100
4	77	22	55	45	=100
5	37	22	15	85	=100

There is one final consideration. It is clear that if we proceed to the next step, a step not taken in Volume 3 to reprice capital itself it follows these capitals will no longer be 100 each because of the variation in their composition of capital. This being so, it implies that 22 in profit will not longer yield a uniform rate of profit of 22%. This is why I titled my article: *PRICES OF PRODUCTION (MUST) YIELD AVERAGE RATES OF PROFIT NOT ON THE OLD CAPITAL BUT THE NEWLY PRICED CAPITAL.*

For this reason the stream of transferred surplus value now needs to be split. One part will be used to convert ‘value of capital’ into ‘priced capital’ via a second transformation while the balance will be used to adjust profits to ensure a uniform rate of profit. In the case of Chapter 9, the amount of capital is 500 or 82% of the total product while 110 or 18% represents the profit element. Thus the 26 in surplus value needing to be redistributed, must be split in two, 21.3% will be used to reprice capital and 4.7% will be used to adjust profits.

This splitting of the surplus value stream has not been done ever since the Transformation Problem surfaced 150 years ago. But it is essential to overcome the incongruity between input and output prices as seen in the tables below taken from the section on Simple Reproduction found in Volume 2. These tables are also used by Von Bortkiewicz. These tables are reproduced below as Table 1.

Table 1 as outlined by the heading ‘Before transformation’ is the original table produced by Marx in Volume 2. Table 2 is ‘After transformation’ yielding the commodity price along the same lines as is found in Chapter 9 of Volume 3 after the equalisation of the rate of profit. Tables 3 and 4 are my solutions and represent fully priced commodity-capital. We are reminded that Dept 1 refers to the department producing means of production while Dept 2 produces article of consumption. There is a mutually useful exchange between the two departments as Department 1 needs articles of consumption to feed its workers and reward its owners, while Dept 1 needs means of production to produce the articles of consumption in the first place.

Table 1.

Dept 1.	4000 c	+ 1000 v	+ 1000 s	= 6000	c/v = 4	P ^l = 20%
Dept 2.	2000 c	+ 1000 v	+ 1000 s	= 4000	c/v = 2	P ^l = 33%
Total	6000 c	+ 2000 v	+ 2000 s	= 10000	c/v = 3	PI = 25%

We note above that the average composition of capital is 3 and the average rate of profit based on a rate of exploitation of 100% yields an average rate profit of 25%. Furthermore there is no disproportionate production between the two department because 1000 v + 1000 s produced in Dept 2 as articles of consumption is exchanged for 2000 c or means of production produced in Dept 1. The only disproportion lies with the rate of profit. Dept 1 with its higher composition of capital enjoys a rate of profit of only 20% compared to 33% in Dept 2 with its lower composition of capital.

In table 2 the redistribution of surplus value results in the harmonising of the rate of profit due to a redistribution of 250 surplus value. The rate of profit enjoyed in both Departments is now 25%.

Table 2.

Dept 1.	4000 c	+ 1000 v	+ 1250 s	= 6250	P ^l = 25%
Dept 2.	2000 c	+ 1000 v	+ 750 s	= 3750	P ^l = 25%
Total	6000 c	+ 2000 v	+ 2000 s	= 10000	PI = 25%

However there is a problem. The exchange relation between Dept 1 and Dept 2 has become lopsided. Dept 1 now receives 2250 from Dept 2 but in return it receives only 2000 back from Dept 1. As a result within a few cycles Dept 2 would become bankrupt as it would lose 250 each time causing reproduction to break down. This is what Von Bortkiewicz sought to overcome with his maths. But by using faulty assumptions because he ignored the law of value, his conclusions are faulty.

However by using the correct assumptions and vectors it is entirely possible to solve the riddle. Here is the solution below. Bearing in mind what was said earlier, we note in Table 1 that 20% of the 10000 product is potential profit while 80% is capital or 8000. It is this 8000 which when priced reprices both inputs and outputs when it is thrown back into production rather than being withdrawn as in the case of s. Furthermore, in Table 2 we note that the total amount of surplus value redistributed amounted to 250. Thus when splitting this stream in the second transformation, only 200 (or 80%) will be used to reprice capital. The other 50 will be used to adjust profits to yield a uniform rate of profit on the repriced capital.

There is a final assumption. We assume 6000 commodities are produced in Dept 1 and 4000 in Dept 2 which means their individual value to begin with is 1. The importance of this will soon become clear. To understand the tables below the following process and assumptions must be understood. Firstly the distribution of 200 to reprice capital must be directed from Dept 2 to Dept 1 due to the lower organic composition found in Dept 1. This is similar to what happened in the first transformation found in Table 2. This means that the capital in Dept 1 will appreciate by 200 to 8200 or by 134 c in the case of the first capital and 66 c in the case of the second (fractions rounded off). But if Dept 1 appreciates by 200 then Dept 2 must depreciate by 200 to balance the movement. Thus total v falls to 1800 v from 2000, with both Departments now registering 900 v down from 1000 v.

In addition we have adjusted the final 50 s in order to yield a rate of profit of 25%. So far so good. However Table 3 is in a holding state where the conditions for pricing inputs takes place. This is similar to the method adopted by Marx later on in Volume 2 in the section on *Expanded Reproduction* (Chapter 21, Part 3, First Illustration). Through dividing 6291 by 6000 units we obtain a preliminary unit price of 1.0485, and dividing 3709 by 4000 units we obtain a preliminary unit price of 0.927. As expected prices have risen for Dept 1 and fallen for Dept 2. We have now arrived at the 'price of commodity' which through the process of circulation forming input prices, will enact the final transformation in Table 4.

To begin we note a disproportion has arisen in Table 3 because v + s in Dept 1 adds up to 2158 c vs 2067 in Dept 2. This, however, is a passing problem which will be resolved in Table 4.

Table 3. (Price of Commodity)

Dept 1.	4133 c	+ 900 v	= (5033 C)	+ 1258 s	6291	P ^l = 25%	Price 1.0485
Dept 2.	2067 c	+ 900 v	= (2967 C)	+ 742 s	3709	P ^l = 25%	Price 0.927
Total	6200 c	+ 1800 v	= (8000 C)	+2000 s	10000	P^l = 25%	Price 1.0
	(1)	(2)	(3)	(4)	(5)	(6)	(7)

In Table 4 this final adjustment is made. The prices found in column (7) above are applied to the interim prices in Table 3. So 4133 c found in column 1 is multiplied by 1.0485 to arrive at 4194 c in Table 4 and so on. Most importantly the value of output has barely changed moving from 6291 to 6288 in in column (5) and inversely from 3709 to 3713 in Dept 2. Thus the circulating prices have changed insignificantly due to rounding off issues by the grand total of 0.03%. (This is the much celebrated equilibrium price.) It would take roughly 250 cycles for reproduction to break down at this rate. Similarly when we add up v + s in Dept 1 it amounts to 2095 vs 2097 c for Department 1, again an immaterial difference. If we were using three rather than four digit sums (as in Chapter 9) that difference would not have even come to light. (For a reconciliation of all the differences please [follow this link.](#))

Table 4. (Price of Production)

Dept 1.	4194 c	+ 928 v	= (5122)	+ 1167 s	6287*	P ^l = 22.8%	Difference -3
Dept 2.	2097 c	+ 927 v	= (3024)	+ 689 s	3713*	P ^l = 22.8%	Difference +3
Total	6291 c	+ 1855 v	= (8146)	1854 s	10000*	P^l = 22.8%	Difference 0
	(1)	(2)	(3)	(4)	(5)	(6)	(7)

(* Rounding off)

This completes the process of converting values into prices. No longer does an incongruity arise because both input and output prices are at their prices of production. No longer does reproduction break down either. What has changed is the rate of profit down from 25% to 22.8%. That is to be expected because some of the surplus value has been capitalised. It means that after the second transformation, while total value and total prices equate, total profits and total surplus value do not.

Does this mean the capitalists are poorer. Not at all. Their capital has grown by 291 while their profits have shrunk by 146 meaning they are better off by 145. This has been made possible because workers share of

the social product measured in prices has fallen from 20% to 18.6%. This is due to the fall in the value of labour power because their articles of consumption have become cheaper. What previously cost 1 now costs only 0.927. However, behind the veil of prices, the actual physical conditions of production and consumption has not changed.

Wonder why it has taken so long for the transformation problem to be solved? Whoever said that capitalism, which is complexed by the chaotic resolution of its inner laws, was easy to explain? Marx has done the working class a great service.

Conclusion.

The rest of the article deals with Von Bortkiewicz adequately. The authors' explanation of the variability of gold which Von Bortkiewicz and his followers freeze in order to reduce the variables they use in their equations down from 4 to 3, is particularly illuminating. I encourage all my readers to investigate this valuable article by James Miller despite its few shortcomings.

The attempts to improve on Chapter 9 has been the graveyard of reputations for the bodgers who have tried and failed. And there are many besides Von Bortkiewicz. I am glad to say that James Miller is not one of them. Quite the contrary, he has deepened our insight into Chapter 9 and the formation of the rate of profit by remaining true to Marx's method.

Brian Green, 2nd May 2024.